

Mathematica 11.3 Integration Test Results

Test results for the 106 problems in "4.4.2.1 (a+b cot)^m (c+d cot)^n.m"

Problem 1: Result more than twice size of optimal antiderivative.

$$\int (a + i a \operatorname{Cot}[c + d x])^n dx$$

Optimal (type 5, 49 leaves, 2 steps):

$$\frac{1}{2 d n} i (a + i a \operatorname{Cot}[c + d x])^n \operatorname{Hypergeometric2F1}\left[1, n, 1 + n, \frac{1}{2} (1 + i \operatorname{Cot}[c + d x])\right]$$

Result (type 5, 112 leaves):

$$\frac{1}{4 d n (1 + n)} i (1 + i \operatorname{Cot}[c + d x])^{-n} (a + i a \operatorname{Cot}[c + d x])^n \left(2 (1 + n) (-1 + (1 + i \operatorname{Cot}[c + d x])^n) + n (1 + i \operatorname{Cot}[c + d x])^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1 + n, 2 + n, \frac{1}{2} (1 + i \operatorname{Cot}[c + d x])\right] \right)$$

Problem 41: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Cot}[x]^2 \sqrt{1 + \operatorname{Cot}[x]} dx$$

Optimal (type 3, 223 leaves, 12 steps):

$$\begin{aligned}
 & -\sqrt{\frac{1}{2}(1+\sqrt{2})} \operatorname{ArcTan}\left[\frac{\sqrt{2(1+\sqrt{2})}-2\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{2(-1+\sqrt{2})}}\right] + \\
 & \sqrt{\frac{1}{2}(1+\sqrt{2})} \operatorname{ArcTan}\left[\frac{\sqrt{2(1+\sqrt{2})}+2\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{2(-1+\sqrt{2})}}\right] - \\
 & \frac{2}{3}(1+\operatorname{Cot}[x])^{3/2} + \frac{\operatorname{Log}[1+\sqrt{2}+\operatorname{Cot}[x]-\sqrt{2(1+\sqrt{2})}\sqrt{1+\operatorname{Cot}[x]}}{2\sqrt{2(1+\sqrt{2})}} - \\
 & \frac{\operatorname{Log}[1+\sqrt{2}+\operatorname{Cot}[x]+\sqrt{2(1+\sqrt{2})}\sqrt{1+\operatorname{Cot}[x]}}{2\sqrt{2(1+\sqrt{2})}}
 \end{aligned}$$

Result (type 3, 69 leaves):

$$-i\sqrt{1-i} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{1-i}}\right] + i\sqrt{1+i} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{1+i}}\right] - \frac{2}{3}(1+\operatorname{Cot}[x])^{3/2}$$

Problem 42: Result unnecessarily involves imaginary or complex numbers.

$$\int \operatorname{Cot}[x] \sqrt{1+\operatorname{Cot}[x]} \, dx$$

Optimal (type 3, 135 leaves, 6 steps):

$$\begin{aligned}
 & \sqrt{\frac{1}{2}(-1+\sqrt{2})} \operatorname{ArcTan}\left[\frac{4-3\sqrt{2}+(2-\sqrt{2})\operatorname{Cot}[x]}{2\sqrt{-7+5\sqrt{2}}\sqrt{1+\operatorname{Cot}[x]}}\right] + \\
 & \sqrt{\frac{1}{2}(1+\sqrt{2})} \operatorname{ArcTanh}\left[\frac{4+3\sqrt{2}+(2+\sqrt{2})\operatorname{Cot}[x]}{2\sqrt{7+5\sqrt{2}}\sqrt{1+\operatorname{Cot}[x]}}\right] - 2\sqrt{1+\operatorname{Cot}[x]}
 \end{aligned}$$

Result (type 3, 61 leaves):

$$\sqrt{1-i} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{1-i}}\right] + \sqrt{1+i} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{1+i}}\right] - 2\sqrt{1+\operatorname{Cot}[x]}$$

Problem 43: Result unnecessarily involves imaginary or complex numbers.

$$\int \cot [x]^2 (1 + \cot [x])^{3/2} dx$$

Optimal (type 3, 139 leaves, 8 steps):

$$-\sqrt{-1 + \sqrt{2}} \operatorname{ArcTan}\left[\frac{3 - 2\sqrt{2} + (1 - \sqrt{2}) \cot [x]}{\sqrt{2(-7 + 5\sqrt{2})} \sqrt{1 + \cot [x]}}\right] -$$

$$\sqrt{1 + \sqrt{2}} \operatorname{ArcTanh}\left[\frac{3 + 2\sqrt{2} + (1 + \sqrt{2}) \cot [x]}{\sqrt{2(7 + 5\sqrt{2})} \sqrt{1 + \cot [x]}}\right] + 2\sqrt{1 + \cot [x]} - \frac{2}{5} (1 + \cot [x])^{5/2}$$

Result (type 3, 96 leaves):

$$\left(\sin [x] \left(-2 \left(\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{1 + \cot [x]}}{\sqrt{1 - i}}\right]}{\sqrt{1 - i}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{1 + \cot [x]}}{\sqrt{1 + i}}\right]}{\sqrt{1 + i}} \right) (1 + \cot [x])^2 \sin [x] - \right. \right.$$

$$\left. \left. \frac{2}{5} (1 + \cot [x])^{5/2} (-5 + 2 \cot [x] + \csc [x]^2) \sin [x] \right) \right) / (\cos [x] + \sin [x])^2$$

Problem 44: Result unnecessarily involves imaginary or complex numbers.

$$\int \cot [x] (1 + \cot [x])^{3/2} dx$$

Optimal (type 3, 221 leaves, 14 steps):

$$\begin{aligned}
 & -\sqrt{1+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2(1+\sqrt{2})}-2\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{2(-1+\sqrt{2})}}\right] + \\
 & \sqrt{1+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2(1+\sqrt{2})}+2\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{2(-1+\sqrt{2})}}\right] - 2\sqrt{1+\operatorname{Cot}[x]} - \\
 & \frac{2}{3}(1+\operatorname{Cot}[x])^{3/2} - \frac{\operatorname{Log}\left[1+\sqrt{2}+\operatorname{Cot}[x]-\sqrt{2(1+\sqrt{2})}\sqrt{1+\operatorname{Cot}[x]}\right]}{2\sqrt{1+\sqrt{2}}} + \\
 & \frac{\operatorname{Log}\left[1+\sqrt{2}+\operatorname{Cot}[x]+\sqrt{2(1+\sqrt{2})}\sqrt{1+\operatorname{Cot}[x]}\right]}{2\sqrt{1+\sqrt{2}}}
 \end{aligned}$$

Result (type 3, 98 leaves):

$$\left(\operatorname{Sin}[x] \left((1+i) \left(-i\sqrt{1-i} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{1-i}}\right] + \sqrt{1+i} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{1+i}}\right] \right) (1+\operatorname{Cot}[x])^2 \operatorname{Sin}[x] - \frac{2}{3}(1+\operatorname{Cot}[x])^{3/2}(4+\operatorname{Cot}[x])(\operatorname{Cos}[x]+\operatorname{Sin}[x]) \right) \right) / (\operatorname{Cos}[x]+\operatorname{Sin}[x])^2$$

Problem 45: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Cot}[x]^2}{\sqrt{1+\operatorname{Cot}[x]}} dx$$

Optimal (type 3, 214 leaves, 12 steps):

$$\begin{aligned}
 & -\frac{1}{2} \sqrt{1+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2(1+\sqrt{2})}-2\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{2(-1+\sqrt{2})}}\right] + \\
 & \frac{1}{2} \sqrt{1+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2(1+\sqrt{2})}+2\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{2(-1+\sqrt{2})}}\right] - \\
 & 2\sqrt{1+\operatorname{Cot}[x]} - \frac{\operatorname{Log}\left[1+\sqrt{2}+\operatorname{Cot}[x]-\sqrt{2(1+\sqrt{2})}\sqrt{1+\operatorname{Cot}[x]}\right]}{4\sqrt{1+\sqrt{2}}} + \\
 & \frac{\operatorname{Log}\left[1+\sqrt{2}+\operatorname{Cot}[x]+\sqrt{2(1+\sqrt{2})}\sqrt{1+\operatorname{Cot}[x]}\right]}{4\sqrt{1+\sqrt{2}}}
 \end{aligned}$$

Result (type 3, 67 leaves):

$$\frac{1}{2} (1-i)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{1-i}}\right] + \frac{1}{2} (1+i)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{1+i}}\right] - 2\sqrt{1+\operatorname{Cot}[x]}$$

Problem 46: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\operatorname{Cot}[x]}{\sqrt{1+\operatorname{Cot}[x]}} dx$$

Optimal (type 3, 121 leaves, 5 steps):

$$\begin{aligned}
 & \frac{1}{2} \sqrt{-1+\sqrt{2}} \operatorname{ArcTan}\left[\frac{3-2\sqrt{2}+(1-\sqrt{2})\operatorname{Cot}[x]}{\sqrt{2(-7+5\sqrt{2})}\sqrt{1+\operatorname{Cot}[x]}}\right] + \\
 & \frac{1}{2} \sqrt{1+\sqrt{2}} \operatorname{ArcTanh}\left[\frac{3+2\sqrt{2}+(1+\sqrt{2})\operatorname{Cot}[x]}{\sqrt{2(7+5\sqrt{2})}\sqrt{1+\operatorname{Cot}[x]}}\right]
 \end{aligned}$$

Result (type 3, 51 leaves):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{1-i}}\right]}{\sqrt{1-i}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{1+i}}\right]}{\sqrt{1+i}}$$

Problem 47: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cot}[x]^2}{(1 + \text{Cot}[x])^{3/2}} dx$$

Optimal (type 3, 139 leaves, 6 steps):

$$\frac{1}{2} \sqrt{\frac{1}{2} (-1 + \sqrt{2})} \text{ArcTan}\left[\frac{4 - 3\sqrt{2} + (2 - \sqrt{2}) \text{Cot}[x]}{2\sqrt{-7 + 5\sqrt{2}} \sqrt{1 + \text{Cot}[x]}}\right] +$$

$$\frac{1}{2} \sqrt{\frac{1}{2} (1 + \sqrt{2})} \text{ArcTanh}\left[\frac{4 + 3\sqrt{2} + (2 + \sqrt{2}) \text{Cot}[x]}{2\sqrt{7 + 5\sqrt{2}} \sqrt{1 + \text{Cot}[x]}}\right] + \frac{1}{\sqrt{1 + \text{Cot}[x]}}$$

Result (type 3, 65 leaves):

$$\frac{1}{2} \sqrt{1 - i} \text{ArcTanh}\left[\frac{\sqrt{1 + \text{Cot}[x]}}{\sqrt{1 - i}}\right] + \frac{1}{2} \sqrt{1 + i} \text{ArcTanh}\left[\frac{\sqrt{1 + \text{Cot}[x]}}{\sqrt{1 + i}}\right] + \frac{1}{\sqrt{1 + \text{Cot}[x]}}$$

Problem 48: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\text{Cot}[x]}{(1 + \text{Cot}[x])^{3/2}} dx$$

Optimal (type 3, 226 leaves, 13 steps):

$$\frac{1}{2} \sqrt{\frac{1}{2} (1 + \sqrt{2})} \text{ArcTan}\left[\frac{\sqrt{2(1 + \sqrt{2}) - 2\sqrt{1 + \text{Cot}[x]}}}{\sqrt{2(-1 + \sqrt{2})}}\right] -$$

$$\frac{1}{2} \sqrt{\frac{1}{2} (1 + \sqrt{2})} \text{ArcTan}\left[\frac{\sqrt{2(1 + \sqrt{2}) + 2\sqrt{1 + \text{Cot}[x]}}}{\sqrt{2(-1 + \sqrt{2})}}\right] -$$

$$\frac{1}{\sqrt{1 + \text{Cot}[x]}} - \frac{\text{Log}[1 + \sqrt{2} + \text{Cot}[x] - \sqrt{2(1 + \sqrt{2})} \sqrt{1 + \text{Cot}[x]}]}{4\sqrt{2(1 + \sqrt{2})}} +$$

$$\frac{\text{Log}[1 + \sqrt{2} + \text{Cot}[x] + \sqrt{2(1 + \sqrt{2})} \sqrt{1 + \text{Cot}[x]}]}{4\sqrt{2(1 + \sqrt{2})}}$$

Result (type 3, 71 leaves):

$$\frac{1}{2} i \sqrt{1-i} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\cot[x]}}{\sqrt{1-i}}\right] - \frac{1}{2} i \sqrt{1+i} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\cot[x]}}{\sqrt{1+i}}\right] - \frac{1}{\sqrt{1+\cot[x]}}$$

Problem 49: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cot[x]^2}{(1+\cot[x])^{5/2}} dx$$

Optimal (type 3, 143 leaves, 8 steps):

$$\frac{1}{4} \sqrt{-1+\sqrt{2}} \operatorname{ArcTan}\left[\frac{3-2\sqrt{2}+(1-\sqrt{2})\cot[x]}{\sqrt{2(-7+5\sqrt{2})}\sqrt{1+\cot[x]}}\right] +$$

$$\frac{1}{4} \sqrt{1+\sqrt{2}} \operatorname{ArcTanh}\left[\frac{3+2\sqrt{2}+(1+\sqrt{2})\cot[x]}{\sqrt{2(7+5\sqrt{2})}\sqrt{1+\cot[x]}}\right] + \frac{1}{3(1+\cot[x])^{3/2}} - \frac{1}{\sqrt{1+\cot[x]}}$$

Result (type 3, 75 leaves):

$$\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{1+\cot[x]}}{\sqrt{1-i}}\right]}{2\sqrt{1-i}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{1+\cot[x]}}{\sqrt{1+i}}\right]}{2\sqrt{1+i}} + \frac{-2-3\cot[x]}{3(1+\cot[x])^{3/2}}$$

Problem 50: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\cot[x]}{(1+\cot[x])^{5/2}} dx$$

Optimal (type 3, 216 leaves, 13 steps):

$$\frac{1}{4} \sqrt{1+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2(1+\sqrt{2})}-2\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{2(-1+\sqrt{2})}}\right] -$$

$$\frac{1}{4} \sqrt{1+\sqrt{2}} \operatorname{ArcTan}\left[\frac{\sqrt{2(1+\sqrt{2})}+2\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{2(-1+\sqrt{2})}}\right] -$$

$$\frac{1}{3(1+\operatorname{Cot}[x])^{3/2}} + \frac{\operatorname{Log}\left[1+\sqrt{2}+\operatorname{Cot}[x]-\sqrt{2(1+\sqrt{2})}\sqrt{1+\operatorname{Cot}[x]}\right]}{8\sqrt{1+\sqrt{2}}} -$$

$$\frac{\operatorname{Log}\left[1+\sqrt{2}+\operatorname{Cot}[x]+\sqrt{2(1+\sqrt{2})}\sqrt{1+\operatorname{Cot}[x]}\right]}{8\sqrt{1+\sqrt{2}}}$$

Result (type 3, 69 leaves):

$$-\frac{1}{4}(1-i)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{1-i}}\right] - \frac{1}{4}(1+i)^{3/2} \operatorname{ArcTanh}\left[\frac{\sqrt{1+\operatorname{Cot}[x]}}{\sqrt{1+i}}\right] - \frac{1}{3(1+\operatorname{Cot}[x])^{3/2}}$$

Problem 75: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e \operatorname{Cot}[c+d x])^{7/2}}{(a+b \operatorname{Cot}[c+d x])^2} dx$$

Optimal (type 3, 437 leaves, 16 steps):

$$\frac{a^{5/2}(3a^2+7b^2)e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b}\sqrt{e \operatorname{Cot}[c+d x]}}{\sqrt{a}\sqrt{e}}\right]}{b^{5/2}(a^2+b^2)^2 d} +$$

$$\frac{(a^2-2ab-b^2)e^{7/2} \operatorname{ArcTan}\left[1-\frac{\sqrt{2}\sqrt{e \operatorname{Cot}[c+d x]}}{\sqrt{e}}\right]}{\sqrt{2}(a^2+b^2)^2 d} - \frac{(a^2-2ab-b^2)e^{7/2} \operatorname{ArcTan}\left[1+\frac{\sqrt{2}\sqrt{e \operatorname{Cot}[c+d x]}}{\sqrt{e}}\right]}{\sqrt{2}(a^2+b^2)^2 d} -$$

$$\frac{(3a^2+2b^2)e^3 \sqrt{e \operatorname{Cot}[c+d x]}}{b^2(a^2+b^2)d} + \frac{a^2 e^2 (e \operatorname{Cot}[c+d x])^{3/2}}{b(a^2+b^2)d(a+b \operatorname{Cot}[c+d x])} + \frac{1}{2\sqrt{2}(a^2+b^2)^2 d}$$

$$(a^2+2ab-b^2)e^{7/2} \operatorname{Log}\left[\sqrt{e}+\sqrt{e} \operatorname{Cot}[c+d x]-\sqrt{2}\sqrt{e \operatorname{Cot}[c+d x]}\right] -$$

$$\frac{1}{2\sqrt{2}(a^2+b^2)^2 d} (a^2+2ab-b^2)e^{7/2} \operatorname{Log}\left[\sqrt{e}+\sqrt{e} \operatorname{Cot}[c+d x]+\sqrt{2}\sqrt{e \operatorname{Cot}[c+d x]}\right]$$

Result (type 3, 775 leaves):

$$\begin{aligned}
 & \left((e \cot [c + d x])^{7/2} \sec [c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])^2 \right. \\
 & \quad \left. \left(-\frac{2}{b^2} - \frac{a^3 \sin [c + d x]}{b^2 (-i a + b) (i a + b) (b \cos [c + d x] + a \sin [c + d x])} \right) \tan [c + d x] \right) / \\
 & \quad \left(d (a + b \cot [c + d x])^2 \right) - \frac{1}{2 (a - i b) (a + i b) b^2 d \cot [c + d x]^{7/2} (a + b \cot [c + d x])^2} \\
 & \quad (e \cot [c + d x])^{7/2} \csc [c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])^2 \\
 & \quad \left(- \left(\left(2 (3 a^3 + 3 a b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] (a + b \cot [c + d x]) \csc [c + d x]^3 \right. \right. \right. \\
 & \quad \left. \left. \left. \sec [c + d x] \right) / \left(\sqrt{a} \sqrt{b} (1 + \cot [c + d x])^2 (b + a \tan [c + d x]) \right) \right) \right) - \\
 & \quad \left(a b^2 \cos [2 (c + d x)] (a + b \cot [c + d x]) \csc [c + d x]^3 \right. \\
 & \quad \left. \left(\frac{4 (a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a - b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] - \right. \right. \\
 & \quad \left. \left. 2 (a - b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}] + (a + b) \left(\log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \right. \right. \right. \\
 & \quad \left. \left. \left. \cot [c + d x]] - \log [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] \right) \right) \right) \sec [c + d x] \right) / \\
 & \quad \frac{(2 (a^2 + b^2) (-1 + \cot [c + d x])^2 (1 + \cot [c + d x])^2 (b + a \tan [c + d x])) - 1}{4 (a^2 + b^2) (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} \\
 & \quad b^3 (a + b \cot [c + d x]) \csc [c + d x]^2 \left(-8 \sqrt{a} \sqrt{b} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] + \sqrt{2} \right. \\
 & \quad \left(-2 (a + b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] + 2 (a + b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}] + \right. \\
 & \quad \left. (a - b) \left(\log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \log [\right. \right. \\
 & \quad \left. \left. \left. 1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] \right) \right) \right) \sec [c + d x]^2 \sin [2 (c + d x)] \right) /
 \end{aligned}$$

Problem 76: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e \operatorname{Cot}[c + d x])^{5/2}}{(a + b \operatorname{Cot}[c + d x])^2} dx$$

Optimal (type 3, 393 leaves, 15 steps):

$$\begin{aligned} & - \frac{a^{3/2} (a^2 + 5 b^2) e^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Cot}[c+dx]}}{\sqrt{a} \sqrt{e}}\right]}{b^{3/2} (a^2 + b^2)^2 d} - \frac{(a^2 + 2 a b - b^2) e^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^2 d} + \\ & \frac{(a^2 + 2 a b - b^2) e^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c+dx]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^2 d} + \frac{a^2 e^2 \sqrt{e \operatorname{Cot}[c + d x]}}{b (a^2 + b^2) d (a + b \operatorname{Cot}[c + d x])} + \\ & \frac{1}{2 \sqrt{2} (a^2 + b^2)^2 d} (a^2 - 2 a b - b^2) e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c + d x] - \sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}\right] - \\ & \frac{1}{2 \sqrt{2} (a^2 + b^2)^2 d} (a^2 - 2 a b - b^2) e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c + d x] + \sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}\right] \end{aligned}$$

Result (type 3, 731 leaves):

$$\begin{aligned}
 & \left(a^2 (e \cot [c + d x])^{5/2} \sec [c + d x] (b \cos [c + d x] + a \sin [c + d x]) \tan [c + d x] \right) / \\
 & \left(b (-i a + b) (i a + b) d (a + b \cot [c + d x])^2 \right) + \\
 & \frac{1}{2 (a - i b) (a + i b) b d \cot [c + d x]^{5/2} (a + b \cot [c + d x])^2} \\
 & (e \cot [c + d x])^{5/2} \csc [c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])^2 \\
 & \left(- \left(\left(2 (a^2 + b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] (a + b \cot [c + d x]) \csc [c + d x]^3 \sec [c + d x] \right) / \right. \right. \\
 & \left. \left. (\sqrt{a} \sqrt{b} (1 + \cot [c + d x])^2 (b + a \tan [c + d x])) \right) - \right. \\
 & \left. \left(b^2 \cos [2 (c + d x)] (a + b \cot [c + d x]) \csc [c + d x]^3 \right. \right. \\
 & \left. \left. \frac{4 (a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a - b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] - \right. \right. \\
 & \left. \left. 2 (a - b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}] + (a + b) (\log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \right. \right. \\
 & \left. \left. \cot [c + d x]] - \log [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]]) \right) \right) \sec [c + d x] \left. \right) / \\
 & \frac{(2 (a^2 + b^2) (-1 + \cot [c + d x])^2 (1 + \cot [c + d x])^2 (b + a \tan [c + d x])) +}{1} \\
 & 4 (a^2 + b^2) (1 + \cot [c + d x])^2 (b + a \tan [c + d x]) \\
 & a b (a + b \cot [c + d x]) \csc [c + d x]^2 \left(-8 \sqrt{a} \sqrt{b} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] + \sqrt{2} \right. \\
 & \left(-2 (a + b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] + 2 (a + b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}] + \right. \\
 & \left. (a - b) (\log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \log [\right. \\
 & \left. \left. 1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x] \right]) \right) \left. \right) \sec [c + d x]^2 \sin [2 (c + d x)] \left. \right)
 \end{aligned}$$

Problem 80: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{(e \cot [c + d x])^{3/2} (a + b \cot [c + d x])^2} dx$$

Optimal (type 3, 437 leaves, 16 steps):

$$\begin{aligned}
 & \frac{b^{5/2} (7 a^2 + 3 b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cot [c+d x]}}{\sqrt{a} \sqrt{e}}\right]}{a^{5/2} (a^2 + b^2)^2 d e^{3/2}} - \\
 & \frac{(a^2 + 2 a b - b^2) \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \cot [c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^2 d e^{3/2}} + \frac{(a^2 + 2 a b - b^2) \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \cot [c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^2 d e^{3/2}} + \\
 & \frac{2 a^2 + 3 b^2}{a^2 (a^2 + b^2) d e \sqrt{e \cot [c+d x]}} - \frac{b^2}{a (a^2 + b^2) d e \sqrt{e \cot [c+d x]} (a + b \cot [c+d x])} + \\
 & \frac{(a^2 - 2 a b - b^2) \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot [c+d x] - \sqrt{2} \sqrt{e \cot [c+d x]}\right]}{2 \sqrt{2} (a^2 + b^2)^2 d e^{3/2}} - \\
 & \frac{(a^2 - 2 a b - b^2) \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot [c+d x] + \sqrt{2} \sqrt{e \cot [c+d x]}\right]}{2 \sqrt{2} (a^2 + b^2)^2 d e^{3/2}}
 \end{aligned}$$

Result (type 3, 773 leaves):

$$\begin{aligned}
 & \left(\cot [c+d x]^2 \csc [c+d x]^2 (b \cos [c+d x]+a \sin [c+d x])^2 \right. \\
 & \quad \left. \left(\frac{b^3 \sin [c+d x]}{a^2 (a^2+b^2) (b \cos [c+d x]+a \sin [c+d x])} + \frac{2 \tan [c+d x]}{a^2} \right) \right) / \\
 & \quad \left(d (e \cot [c+d x])^{3/2} (a+b \cot [c+d x])^2 \right) - \\
 & \quad \frac{1}{2 a^2 (-i a+b) (i a+b) d (e \cot [c+d x])^{3/2} (a+b \cot [c+d x])^2} \\
 & \quad \cot [c+d x]^{3/2} \csc [c+d x]^2 (b \cos [c+d x]+a \sin [c+d x])^2 \\
 & \quad \left(- \left(\left(2 (3 a^2 b+3 b^3) \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c+d x]}}{\sqrt{a}} \right] (a+b \cot [c+d x]) \csc [c+d x]^3 \right. \right. \right. \\
 & \quad \left. \left. \left. \sec [c+d x] \right) \right) / \left(\sqrt{a} \sqrt{b} (1+\cot [c+d x])^2 (b+a \tan [c+d x]) \right) \right) + \\
 & \quad \left(a^2 b \cos [2 (c+d x)] (a+b \cot [c+d x]) \csc [c+d x]^3 \right. \\
 & \quad \left(\frac{4 (a^2-b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c+d x]}}{\sqrt{a}} \right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a-b) \operatorname{ArcTan} [1-\sqrt{2} \sqrt{\cot [c+d x]}] - \right. \\
 & \quad \left. 2 (a-b) \operatorname{ArcTan} [1+\sqrt{2} \sqrt{\cot [c+d x]}] + (a+b) (\log [1-\sqrt{2} \sqrt{\cot [c+d x]} + \right. \\
 & \quad \left. \left. \left. \cot [c+d x] - \log [1+\sqrt{2} \sqrt{\cot [c+d x]} + \cot [c+d x]] \right) \right) \right) \sec [c+d x] \right) / \\
 & \quad \frac{1}{2 (a^2+b^2) (-1+\cot [c+d x])^2 (1+\cot [c+d x])^2 (b+a \tan [c+d x])} - \\
 & \quad \frac{1}{4 (a^2+b^2) (1+\cot [c+d x])^2 (b+a \tan [c+d x])} \\
 & \quad a^3 (a+b \cot [c+d x]) \csc [c+d x]^2 \left(-8 \sqrt{a} \sqrt{b} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c+d x]}}{\sqrt{a}} \right] + \sqrt{2} \right. \\
 & \quad \left(-2 (a+b) \operatorname{ArcTan} [1-\sqrt{2} \sqrt{\cot [c+d x]}] + 2 (a+b) \operatorname{ArcTan} [1+\sqrt{2} \sqrt{\cot [c+d x]}] + \right. \\
 & \quad \left. (a-b) (\log [1-\sqrt{2} \sqrt{\cot [c+d x]} + \cot [c+d x]] - \log [\right. \\
 & \quad \left. \left. \left. 1+\sqrt{2} \sqrt{\cot [c+d x]} + \cot [c+d x] \right) \right) \right) \sec [c+d x]^2 \sin [2 (c+d x)] \right) /
 \end{aligned}$$

Problem 81: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e \operatorname{Cot}[c + d x])^{9/2}}{(a + b \operatorname{Cot}[c + d x])^3} dx$$

Optimal (type 3, 529 leaves, 17 steps):

$$\frac{a^{5/2} (15 a^4 + 46 a^2 b^2 + 63 b^4) e^{9/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Cot}[c + d x]}}{\sqrt{a} \sqrt{e}}\right]}{4 b^{7/2} (a^2 + b^2)^3 d} +$$

$$\frac{(a - b) (a^2 + 4 a b + b^2) e^{9/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} -$$

$$\frac{(a - b) (a^2 + 4 a b + b^2) e^{9/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} -$$

$$\frac{(15 a^4 + 31 a^2 b^2 + 8 b^4) e^4 \sqrt{e \operatorname{Cot}[c + d x]}}{4 b^3 (a^2 + b^2)^2 d} + \frac{a^2 e^2 (e \operatorname{Cot}[c + d x])^{5/2}}{2 b (a^2 + b^2) d (a + b \operatorname{Cot}[c + d x])^2} +$$

$$\frac{a^2 (5 a^2 + 13 b^2) e^3 (e \operatorname{Cot}[c + d x])^{3/2}}{4 b^2 (a^2 + b^2)^2 d (a + b \operatorname{Cot}[c + d x])} - \frac{1}{2 \sqrt{2} (a^2 + b^2)^3 d}$$

$$(a + b) (a^2 - 4 a b + b^2) e^{9/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c + d x] - \sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}\right] +$$

$$\frac{1}{2 \sqrt{2} (a^2 + b^2)^3 d} (a + b) (a^2 - 4 a b + b^2) e^{9/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c + d x] + \sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}\right]$$

Result (type 3, 897 leaves):

$$\begin{aligned}
 & \left((e \cot [c + d x])^{9/2} \sec [c + d x]^3 (b \cos [c + d x] + a \sin [c + d x])^3 \right. \\
 & \quad \left(-\frac{5 a^4 + 8 a^2 b^2 + 4 b^4}{2 b^3 (-i a + b)^2 (i a + b)^2} + \frac{a^4}{2 b (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])^2} + \right. \\
 & \quad \left. \frac{-5 a^5 \sin [c + d x] - 17 a^3 b^2 \sin [c + d x]}{4 b^3 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])^2} \right) \tan [c + d x] \Bigg/ \\
 & \quad \left(d (a + b \cot [c + d x])^3 \right) - \frac{1}{8 (a - i b)^2 (a + i b)^2 b^3 d \cot [c + d x]^{9/2} (a + b \cot [c + d x])^3} \\
 & \quad (e \cot [c + d x])^{9/2} \csc [c + d x]^3 (b \cos [c + d x] + a \sin [c + d x])^3 \\
 & \quad \left(-\left(\left(2 (15 a^5 + 31 a^3 b^2 + 16 a b^4) \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] (a + b \cot [c + d x]) \right. \right. \right. \\
 & \quad \left. \left. \left. \csc [c + d x]^3 \sec [c + d x] \right) \right) / \left(\sqrt{a} \sqrt{b} (1 + \cot [c + d x])^2 (b + a \tan [c + d x]) \right) \right) - \\
 & \quad \left(\frac{1}{(a^2 + b^2) (-1 + \cot [c + d x])^2 (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} \right) \\
 & \quad 4 a b^4 \cos [2 (c + d x)] (a + b \cot [c + d x]) \csc [c + d x]^3 \\
 & \quad \left(\frac{4 (a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a - b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] - \right. \\
 & \quad \left. 2 (a - b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}] + (a + b) (\log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \right. \\
 & \quad \left. \cot [c + d x]] - \log [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]]) \right) \Bigg) \sec [c + d x] - \\
 & \quad \frac{1}{4 (a^2 + b^2) (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} (-4 a^2 b^3 + 4 b^5) (a + b \cot [c + d x]) \\
 & \quad \csc [c + d x]^2 \left(-8 \sqrt{a} \sqrt{b} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] + \sqrt{2} \right. \\
 & \quad \left(-2 (a + b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] + 2 (a + b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}] + \right. \\
 & \quad \left. (a - b) (\log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \log [\right. \\
 & \quad \left. \left. 1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x] \right] \right) \Bigg) \sec [c + d x]^2 \sin [2 (c + d x)] \Bigg)
 \end{aligned}$$

Problem 82: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e \operatorname{Cot}[c + d x])^{7/2}}{(a + b \operatorname{Cot}[c + d x])^3} dx$$

Optimal (type 3, 476 leaves, 16 steps):

$$\begin{aligned} & - \frac{a^{3/2} (3 a^4 + 6 a^2 b^2 + 35 b^4) e^{7/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Cot}[c+d x]}}{\sqrt{a} \sqrt{e}}\right]}{4 b^{5/2} (a^2 + b^2)^3 d} + \\ & \frac{(a + b) (a^2 - 4 a b + b^2) e^{7/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} - \\ & \frac{(a + b) (a^2 - 4 a b + b^2) e^{7/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} + \\ & \frac{a^2 e^2 (e \operatorname{Cot}[c + d x])^{3/2}}{2 b (a^2 + b^2) d (a + b \operatorname{Cot}[c + d x])^2} + \frac{a^2 (3 a^2 + 11 b^2) e^3 \sqrt{e \operatorname{Cot}[c + d x]}}{4 b^2 (a^2 + b^2)^2 d (a + b \operatorname{Cot}[c + d x])} + \frac{1}{2 \sqrt{2} (a^2 + b^2)^3 d} \\ & (a - b) (a^2 + 4 a b + b^2) e^{7/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c + d x] - \sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}\right] - \\ & \frac{1}{2 \sqrt{2} (a^2 + b^2)^3 d} (a - b) (a^2 + 4 a b + b^2) e^{7/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c + d x] + \sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}\right] \end{aligned}$$

Result (type 3, 870 leaves):

$$\begin{aligned}
 & \left((e \cot [c + d x])^{7/2} \sec [c + d x]^3 (b \cos [c + d x] + a \sin [c + d x])^3 \right. \\
 & \quad \left(\frac{a^3}{2 b^2 (-i a + b)^2 (i a + b)^2} - \frac{a^3}{2 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])^2} + \right. \\
 & \quad \left. \frac{a^4 \sin [c + d x] + 13 a^2 b^2 \sin [c + d x]}{4 b^2 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])} \right) \Big/ (d (a + b \cot [c + d x])^3) + \\
 & \quad \frac{1}{8 (a - i b)^2 (a + i b)^2 b^2 d \cot [c + d x]^{7/2} (a + b \cot [c + d x])^3} \\
 & \quad (e \cot [c + d x])^{7/2} \csc [c + d x]^3 (b \cos [c + d x] + a \sin [c + d x])^3 \\
 & \quad \left(- \left(\left(2 (3 a^4 + 7 a^2 b^2 + 4 b^4) \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] (a + b \cot [c + d x]) \csc [c + d x]^3 \right. \right. \right. \\
 & \quad \left. \left. \left. \sec [c + d x] \right) \Big/ (\sqrt{a} \sqrt{b} (1 + \cot [c + d x])^2 (b + a \tan [c + d x])) \right) - \right. \\
 & \quad \left((-4 a^2 b^2 + 4 b^4) \cos [2 (c + d x)] (a + b \cot [c + d x]) \csc [c + d x]^3 \right. \\
 & \quad \left(\frac{4 (a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a - b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] - \right. \\
 & \quad \left. 2 (a - b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}] + (a + b) (\log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \right. \\
 & \quad \left. \left. \left. \cot [c + d x] - \log [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]]) \right) \right) \sec [c + d x] \Big/ \\
 & \quad \frac{2 (a^2 + b^2) (-1 + \cot [c + d x])^2 (1 + \cot [c + d x])^2 (b + a \tan [c + d x])}{1} + \\
 & \quad \frac{(a^2 + b^2) (1 + \cot [c + d x])^2 (b + a \tan [c + d x])}{2 a b^3 (a + b \cot [c + d x]) \csc [c + d x]^2} \left(-8 \sqrt{a} \sqrt{b} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] + \sqrt{2} \right. \\
 & \quad \left(-2 (a + b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] + 2 (a + b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}] + \right. \\
 & \quad \left. (a - b) (\log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \log [\right. \\
 & \quad \left. \left. \left. 1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x] \right) \right) \right) \sec [c + d x]^2 \sin [2 (c + d x)] \Big/
 \end{aligned}$$

Problem 83: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e \operatorname{Cot}[c + d x])^{5/2}}{(a + b \operatorname{Cot}[c + d x])^3} dx$$

Optimal (type 3, 470 leaves, 16 steps):

$$\begin{aligned} & - \frac{\sqrt{a} (a^4 + 18 a^2 b^2 - 15 b^4) e^{5/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Cot}[c+d x]}}{\sqrt{a} \sqrt{e}}\right]}{4 b^{3/2} (a^2 + b^2)^3 d} \\ & + \frac{(a - b) (a^2 + 4 a b + b^2) e^{5/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} \\ & + \frac{(a - b) (a^2 + 4 a b + b^2) e^{5/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} \\ & - \frac{a^2 e^2 \sqrt{e \operatorname{Cot}[c + d x]}}{2 b (a^2 + b^2) d (a + b \operatorname{Cot}[c + d x])^2} - \frac{a (a^2 + 9 b^2) e^2 \sqrt{e \operatorname{Cot}[c + d x]}}{4 b (a^2 + b^2)^2 d (a + b \operatorname{Cot}[c + d x])} + \frac{1}{2 \sqrt{2} (a^2 + b^2)^3 d} \\ & - \frac{(a + b) (a^2 - 4 a b + b^2) e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c + d x] - \sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}\right]}{2 \sqrt{2} (a^2 + b^2)^3 d} \\ & - \frac{1}{2 \sqrt{2} (a^2 + b^2)^3 d} (a + b) (a^2 - 4 a b + b^2) e^{5/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c + d x] + \sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}\right] \end{aligned}$$

Result (type 3, 864 leaves):

$$\begin{aligned}
 & \left((e \cot [c + d x])^{5/2} \operatorname{Csc}[c + d x] \operatorname{Sec}[c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])^3 \right. \\
 & \left. \left(-\frac{a^2}{2 b (-i a + b)^2 (i a + b)^2} + \frac{a^2 b}{2 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])^2} - \right. \right. \\
 & \left. \left. \frac{3 (-a^3 \sin [c + d x] + 3 a b^2 \sin [c + d x])}{4 b (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])} \right) \right) / (d (a + b \cot [c + d x])^3) + \\
 & \frac{1}{8 (a - i b)^2 (a + i b)^2 b d \cot [c + d x]^{5/2} (a + b \cot [c + d x])^3} \\
 & (e \cot [c + d x])^{5/2} \operatorname{Csc}[c + d x]^3 (b \cos [c + d x] + a \sin [c + d x])^3 \\
 & \left(-\left(\left(2 (a^3 + a b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}}\right] (a + b \cot [c + d x]) \operatorname{Csc}[c + d x]^3 \operatorname{Sec}[c + d x] \right) / \right. \right. \\
 & \left. \left. (\sqrt{a} \sqrt{b} (1 + \cot [c + d x])^2 (b + a \tan [c + d x])) \right) - \right. \\
 & \left. (1 / ((a^2 + b^2) (-1 + \cot [c + d x])^2 (1 + \cot [c + d x])^2 (b + a \tan [c + d x]))) \right. \\
 & \left. 4 a b^2 \cos [2 (c + d x)] (a + b \cot [c + d x]) \operatorname{Csc}[c + d x]^3 \right. \\
 & \left. \left(\frac{4 (a^2 - b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}}\right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a - b) \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{\cot [c + d x]}] - \right. \right. \\
 & \left. \left. 2 (a - b) \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{\cot [c + d x]}] + (a + b) (\operatorname{Log}[1 - \sqrt{2} \sqrt{\cot [c + d x]} + \right. \right. \\
 & \left. \left. \cot [c + d x]] - \operatorname{Log}[1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]]) \right) \right) \operatorname{Sec}[c + d x] - \\
 & \frac{1}{4 (a^2 + b^2) (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} (-4 a^2 b + 4 b^3) (a + b \cot [c + d x]) \\
 & \operatorname{Csc}[c + d x]^2 \left(-8 \sqrt{a} \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}}\right] + \sqrt{2} \right. \\
 & \left. (-2 (a + b) \operatorname{ArcTan}[1 - \sqrt{2} \sqrt{\cot [c + d x]}] + 2 (a + b) \operatorname{ArcTan}[1 + \sqrt{2} \sqrt{\cot [c + d x]}] + \right. \\
 & \left. (a - b) (\operatorname{Log}[1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \operatorname{Log}[\right. \\
 & \left. \left. 1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x] \right]) \right) \operatorname{Sec}[c + d x]^2 \sin [2 (c + d x)] \left. \right)
 \end{aligned}$$

Problem 84: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{(e \cot [c + d x])^{3/2}}{(a + b \cot [c + d x])^3} dx$$

Optimal (type 3, 461 leaves, 16 steps):

$$\begin{aligned} & - \frac{(3 a^4 - 26 a^2 b^2 + 3 b^4) e^{3/2} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cot [c+d x]}}{\sqrt{a} \sqrt{e}}\right]}{4 \sqrt{a} \sqrt{b} (a^2 + b^2)^3 d} - \\ & \frac{(a + b) (a^2 - 4 a b + b^2) e^{3/2} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \cot [c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} + \\ & \frac{(a + b) (a^2 - 4 a b + b^2) e^{3/2} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \cot [c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} - \\ & \frac{a e \sqrt{e \cot [c + d x]}}{2 (a^2 + b^2) d (a + b \cot [c + d x])^2} - \frac{(3 a^2 - 5 b^2) e \sqrt{e \cot [c + d x]}}{4 (a^2 + b^2)^2 d (a + b \cot [c + d x])} - \frac{1}{2 \sqrt{2} (a^2 + b^2)^3 d} \\ & (a - b) (a^2 + 4 a b + b^2) e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e \cot [c + d x]} - \sqrt{2} \sqrt{e \cot [c + d x]}\right] + \\ & \frac{1}{2 \sqrt{2} (a^2 + b^2)^3 d} (a - b) (a^2 + 4 a b + b^2) e^{3/2} \operatorname{Log}\left[\sqrt{e} + \sqrt{e \cot [c + d x]} + \sqrt{2} \sqrt{e \cot [c + d x]}\right] \end{aligned}$$

Result (type 3, 851 leaves):

$$\begin{aligned}
 & \left((e \cot [c + d x])^{3/2} \operatorname{Csc}[c + d x]^2 \operatorname{Sec}[c + d x] (b \cos [c + d x] + a \sin [c + d x])^3 \right. \\
 & \quad \left(\frac{a}{2 (-i a + b)^2 (i a + b)^2} - \frac{a b^2}{2 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])^2} + \right. \\
 & \quad \left. \frac{-7 a^2 \sin [c + d x] + 5 b^2 \sin [c + d x]}{4 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])} \right) \Bigg) / (d (a + b \cot [c + d x])^3) + \\
 & \frac{1}{8 (a - i b)^2 (a + i b)^2 d \cot [c + d x]^{3/2} (a + b \cot [c + d x])^3} \\
 & (e \cot [c + d x])^{3/2} \operatorname{Csc}[c + d x]^3 (b \cos [c + d x] + a \sin [c + d x])^3 \\
 & \left(- \left(\left(2 (-a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] (a + b \cot [c + d x]) \operatorname{Csc}[c + d x]^3 \operatorname{Sec}[c + d x] \right) / \right. \right. \\
 & \quad \left. \left. (\sqrt{a} \sqrt{b} (1 + \cot [c + d x])^2 (b + a \tan [c + d x])) \right) \right) - \\
 & \left((4 a^2 - 4 b^2) \cos [2 (c + d x)] (a + b \cot [c + d x]) \operatorname{Csc}[c + d x]^3 \right. \\
 & \quad \left(\frac{4 (a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a - b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] - \right. \\
 & \quad \left. 2 (a - b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}] + (a + b) (\log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \right. \\
 & \quad \left. \cot [c + d x]] - \log [1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]]) \right) \operatorname{Sec}[c + d x] \Bigg) / \\
 & \frac{1}{2 (a^2 + b^2) (-1 + \cot [c + d x])^2 (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} - \\
 & \frac{1}{(a^2 + b^2) (1 + \cot [c + d x])^2 (b + a \tan [c + d x])} \\
 & 2 a b (a + b \cot [c + d x]) \operatorname{Csc}[c + d x]^2 \left(-8 \sqrt{a} \sqrt{b} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c + d x]}}{\sqrt{a}} \right] + \sqrt{2} \right. \\
 & \quad \left(-2 (a + b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c + d x]}] + 2 (a + b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c + d x]}] + \right. \\
 & \quad \left. (a - b) (\log [1 - \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] - \log [\right. \\
 & \quad \left. \left. 1 + \sqrt{2} \sqrt{\cot [c + d x]} + \cot [c + d x]] \right) \right) \operatorname{Sec}[c + d x]^2 \sin [2 (c + d x)] \Bigg)
 \end{aligned}$$

Problem 85: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{\sqrt{e \cot [c+d x]}}{(a+b \cot [c+d x])^3} dx$$

Optimal (type 3, 463 leaves, 16 steps):

$$\begin{aligned} & \frac{\sqrt{b} (15 a^4 - 18 a^2 b^2 - b^4) \sqrt{e} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cot [c+d x]}}{\sqrt{a} \sqrt{e}}\right]}{4 a^{3/2} (a^2 + b^2)^3 d} + \\ & \frac{(a-b) (a^2 + 4 a b + b^2) \sqrt{e} \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \cot [c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} - \\ & \frac{(a-b) (a^2 + 4 a b + b^2) \sqrt{e} \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \cot [c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d} + \\ & \frac{b \sqrt{e \cot [c+d x]}}{2 (a^2 + b^2) d (a+b \cot [c+d x])^2} + \frac{b (7 a^2 - b^2) \sqrt{e \cot [c+d x]}}{4 a (a^2 + b^2)^2 d (a+b \cot [c+d x])} - \frac{1}{2 \sqrt{2} (a^2 + b^2)^3 d} \\ & (a+b) (a^2 - 4 a b + b^2) \sqrt{e} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot [c+d x] - \sqrt{2} \sqrt{e \cot [c+d x]}\right] + \\ & \frac{1}{2 \sqrt{2} (a^2 + b^2)^3 d} (a+b) (a^2 - 4 a b + b^2) \sqrt{e} \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot [c+d x] + \sqrt{2} \sqrt{e \cot [c+d x]}\right] \end{aligned}$$

Result (type 3, 852 leaves):

$$\begin{aligned}
 & \left(\sqrt{e \cot [c+d x]} \operatorname{Csc}[c+d x]^3 (b \cos [c+d x]+a \sin [c+d x])^3 \right. \\
 & \quad \left(-\frac{b}{2(-i a+b)^2(i a+b)^2} + \frac{b^3}{2(-i a+b)^2(i a+b)^2(b \cos [c+d x]+a \sin [c+d x])^2} + \right. \\
 & \quad \left. \left. \frac{11 a^2 b \sin [c+d x]-b^3 \sin [c+d x]}{4 a(-i a+b)^2(i a+b)^2(b \cos [c+d x]+a \sin [c+d x])} \right) \right) / (d(a+b \cot [c+d x])^3) + \\
 & \frac{1}{8 a(a-i b)^2(a+i b)^2 d \sqrt{\cot [c+d x]}(a+b \cot [c+d x])^3} \\
 & \sqrt{e \cot [c+d x]} \operatorname{Csc}[c+d x]^3 (b \cos [c+d x]+a \sin [c+d x])^3 \\
 & \left(-\left(\left(2\left(a^2 b+b^3\right) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot [c+d x]}}{\sqrt{a}}\right](a+b \cot [c+d x]) \operatorname{Csc}[c+d x]^3 \operatorname{Sec}[c+d x] \right) \right) / \right. \\
 & \quad \left. \left(\sqrt{a} \sqrt{b}\left(1+\cot [c+d x]\right)^2(b+a \tan [c+d x]) \right) \right) + \\
 & \left(1 / \left(\left(a^2+b^2\right)\left(-1+\cot [c+d x]\right)^2\left(1+\cot [c+d x]\right)^2(b+a \tan [c+d x]) \right) \right) \\
 & 4 a^2 b \cos [2(c+d x)](a+b \cot [c+d x]) \operatorname{Csc}[c+d x]^3 \\
 & \left(\frac{4\left(a^2-b^2\right) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot [c+d x]}}{\sqrt{a}}\right]}{\sqrt{a} \sqrt{b}} + \sqrt{2}\left(2(a-b) \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\cot [c+d x]}\right]- \right. \right. \\
 & \quad \left. \left. 2(a-b) \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\cot [c+d x]}\right]+(a+b)\left(\operatorname{Log}\left[1-\sqrt{2} \sqrt{\cot [c+d x]}\right]+ \right. \right. \\
 & \quad \left. \left. \cot [c+d x]\right)-\operatorname{Log}\left[1+\sqrt{2} \sqrt{\cot [c+d x]}+\cot [c+d x]\right]\right) \right) \operatorname{Sec}[c+d x]- \\
 & \frac{1}{4\left(a^2+b^2\right)\left(1+\cot [c+d x]\right)^2(b+a \tan [c+d x])}\left(4 a^3-4 a b^2\right)(a+b \cot [c+d x]) \\
 & \operatorname{Csc}[c+d x]^2\left(-8 \sqrt{a} \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot [c+d x]}}{\sqrt{a}}\right]+ \sqrt{2}\right. \\
 & \quad \left(-2(a+b) \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\cot [c+d x]}\right]+2(a+b) \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\cot [c+d x]}\right]+ \right. \\
 & \quad \left.(a-b)\left(\operatorname{Log}\left[1-\sqrt{2} \sqrt{\cot [c+d x]}+\cot [c+d x]\right)-\operatorname{Log}\left[\right. \right. \\
 & \quad \left. \left. 1+\sqrt{2} \sqrt{\cot [c+d x]}+\cot [c+d x]\right]\right) \right) \operatorname{Sec}[c+d x]^2 \sin [2(c+d x)] \left. \right)
 \end{aligned}$$

Problem 86: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\sqrt{e \cot [c+d x]} (a+b \cot [c+d x])^3} dx$$

Optimal (type 3, 476 leaves, 16 steps):

$$\begin{aligned} & - \frac{b^{3/2} (35 a^4 + 6 a^2 b^2 + 3 b^4) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \cot [c+d x]}}{\sqrt{a} \sqrt{e}}\right]}{4 a^{5/2} (a^2 + b^2)^3 d \sqrt{e}} + \\ & \frac{(a+b) (a^2 - 4 a b + b^2) \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \cot [c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d \sqrt{e}} - \\ & \frac{(a+b) (a^2 - 4 a b + b^2) \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \cot [c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d \sqrt{e}} - \\ & \frac{b^2 \sqrt{e \cot [c+d x]}}{2 a (a^2 + b^2) d e (a+b \cot [c+d x])^2} - \frac{b^2 (11 a^2 + 3 b^2) \sqrt{e \cot [c+d x]}}{4 a^2 (a^2 + b^2)^2 d e (a+b \cot [c+d x])} + \\ & \left((a-b) (a^2 + 4 a b + b^2) \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot [c+d x] - \sqrt{2} \sqrt{e \cot [c+d x]}\right] \right) / \\ & \left(2 \sqrt{2} (a^2 + b^2)^3 d \sqrt{e} \right) - \\ & \left((a-b) (a^2 + 4 a b + b^2) \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \cot [c+d x] + \sqrt{2} \sqrt{e \cot [c+d x]}\right] \right) / \\ & \left(2 \sqrt{2} (a^2 + b^2)^3 d \sqrt{e} \right) \end{aligned}$$

Result (type 3, 879 leaves):

$$\begin{aligned}
 & \left(\cot [c+d x] \csc [c+d x]^3 (b \cos [c+d x] + a \sin [c+d x])^3 \right. \\
 & \quad \left(\frac{b^2}{2 a (-i a+b)^2 (i a+b)^2} - \frac{b^4}{2 a (-i a+b)^2 (i a+b)^2 (b \cos [c+d x] + a \sin [c+d x])^2} - \right. \\
 & \quad \left. \frac{3 (5 a^2 b^2 \sin [c+d x] + b^4 \sin [c+d x])}{4 a^2 (-i a+b)^2 (i a+b)^2 (b \cos [c+d x] + a \sin [c+d x])} \right) \Bigg) / \\
 & \quad \left(d \sqrt{e \cot [c+d x]} (a+b \cot [c+d x])^3 \right) - \\
 & \quad \frac{1}{8 a^2 (a-i b)^2 (a+i b)^2 d \sqrt{e \cot [c+d x]} (a+b \cot [c+d x])^3} \\
 & \quad \sqrt{\cot [c+d x]} \csc [c+d x]^3 (b \cos [c+d x] + a \sin [c+d x])^3 \\
 & \quad \left(- \left(\left(2 (-4 a^4 - 7 a^2 b^2 - 3 b^4) \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c+d x]}}{\sqrt{a}} \right] (a+b \cot [c+d x]) \csc [c+d x]^3 \right. \right. \right. \\
 & \quad \left. \left. \left. \operatorname{Sec} [c+d x] \right) / \left(\sqrt{a} \sqrt{b} (1+\cot [c+d x])^2 (b+a \tan [c+d x]) \right) \right) \right) - \\
 & \quad \left(4 a^4 - 4 a^2 b^2 \right) \cos [2 (c+d x)] (a+b \cot [c+d x]) \csc [c+d x]^3 \\
 & \quad \left(\frac{4 (a^2 - b^2) \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c+d x]}}{\sqrt{a}} \right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2 (a-b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c+d x]}] - \right. \\
 & \quad \left. 2 (a-b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c+d x]}] + (a+b) \left(\log [1 - \sqrt{2} \sqrt{\cot [c+d x]} + \right. \right. \\
 & \quad \left. \left. \cot [c+d x]] - \log [1 + \sqrt{2} \sqrt{\cot [c+d x]} + \cot [c+d x]] \right) \right) \operatorname{Sec} [c+d x] \Bigg) / \\
 & \quad \frac{2 (a^2 + b^2) (-1 + \cot [c+d x])^2 (1 + \cot [c+d x])^2 (b+a \tan [c+d x])}{1} - \\
 & \quad \frac{(a^2 + b^2) (1 + \cot [c+d x])^2 (b+a \tan [c+d x])}{2 a^3 b (a+b \cot [c+d x]) \csc [c+d x]^2} \left(-8 \sqrt{a} \sqrt{b} \operatorname{ArcTan} \left[\frac{\sqrt{b} \sqrt{\cot [c+d x]}}{\sqrt{a}} \right] + \sqrt{2} \right. \\
 & \quad \left(-2 (a+b) \operatorname{ArcTan} [1 - \sqrt{2} \sqrt{\cot [c+d x]}] + 2 (a+b) \operatorname{ArcTan} [1 + \sqrt{2} \sqrt{\cot [c+d x]}] + \right. \\
 & \quad \left. (a-b) \left(\log [1 - \sqrt{2} \sqrt{\cot [c+d x]} + \cot [c+d x]] - \log [\right. \right. \\
 & \quad \left. \left. 1 + \sqrt{2} \sqrt{\cot [c+d x]} + \cot [c+d x]] \right) \right) \operatorname{Sec} [c+d x]^2 \sin [2 (c+d x)] \Bigg)
 \end{aligned}$$

Problem 87: Result unnecessarily involves imaginary or complex numbers.

$$\int \frac{1}{\left(e \operatorname{Cot}[c + d x] \right)^{3/2} \left(a + b \operatorname{Cot}[c + d x] \right)^3} dx$$

Optimal (type 3, 529 leaves, 17 steps):

$$\frac{b^{5/2} (63 a^4 + 46 a^2 b^2 + 15 b^4) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{e \operatorname{Cot}[c+d x]}}{\sqrt{a} \sqrt{e}}\right]}{4 a^{7/2} (a^2 + b^2)^3 d e^{3/2}} -$$

$$\frac{(a - b) (a^2 + 4 a b + b^2) \operatorname{ArcTan}\left[1 - \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d e^{3/2}} +$$

$$\frac{(a - b) (a^2 + 4 a b + b^2) \operatorname{ArcTan}\left[1 + \frac{\sqrt{2} \sqrt{e \operatorname{Cot}[c+d x]}}{\sqrt{e}}\right]}{\sqrt{2} (a^2 + b^2)^3 d e^{3/2}} + \frac{8 a^4 + 31 a^2 b^2 + 15 b^4}{4 a^3 (a^2 + b^2)^2 d e \sqrt{e \operatorname{Cot}[c + d x]}} -$$

$$\frac{b^2}{2 a (a^2 + b^2) d e \sqrt{e \operatorname{Cot}[c + d x]} (a + b \operatorname{Cot}[c + d x])^2} -$$

$$\frac{b^2 (13 a^2 + 5 b^2)}{4 a^2 (a^2 + b^2)^2 d e \sqrt{e \operatorname{Cot}[c + d x]} (a + b \operatorname{Cot}[c + d x])} +$$

$$\left((a + b) (a^2 - 4 a b + b^2) \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c + d x] - \sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}\right] \right) /$$

$$\left(2 \sqrt{2} (a^2 + b^2)^3 d e^{3/2} \right) -$$

$$\left((a + b) (a^2 - 4 a b + b^2) \operatorname{Log}\left[\sqrt{e} + \sqrt{e} \operatorname{Cot}[c + d x] + \sqrt{2} \sqrt{e \operatorname{Cot}[c + d x]}\right] \right) /$$

$$\left(2 \sqrt{2} (a^2 + b^2)^3 d e^{3/2} \right)$$

Result (type 3, 894 leaves):

$$\begin{aligned}
 & \left(\cot [c+d x]^2 \csc [c+d x]^3 (b \cos [c+d x]+a \sin [c+d x])^3 \right. \\
 & \quad \left(-\frac{b^3}{2 a^2 (-i a+b)^2 (i a+b)^2} + \frac{b^5}{2 a^2 (-i a+b)^2 (i a+b)^2 (b \cos [c+d x]+a \sin [c+d x])^2} + \right. \\
 & \quad \left. \frac{19 a^2 b^3 \sin [c+d x]+7 b^5 \sin [c+d x]}{4 a^3 (-i a+b)^2 (i a+b)^2 (b \cos [c+d x]+a \sin [c+d x])} + \frac{2 \tan [c+d x]}{a^3} \right) \Bigg) / \\
 & \quad \left(d (e \cot [c+d x])^{3/2} (a+b \cot [c+d x])^3 \right) - \\
 & \quad \frac{1}{8 a^3 (a-i b)^2 (a+i b)^2 d (e \cot [c+d x])^{3/2} (a+b \cot [c+d x])^3} \\
 & \quad \cot [c+d x]^{3/2} \csc [c+d x]^3 (b \cos [c+d x]+a \sin [c+d x])^3 \\
 & \quad \left(-\left(\left(2 (16 a^4 b+31 a^2 b^3+15 b^5) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot [c+d x]}}{\sqrt{a}}\right] (a+b \cot [c+d x]) \right. \right. \right. \\
 & \quad \left. \left. \left. \csc [c+d x]^3 \sec [c+d x] \right) / \left(\sqrt{a} \sqrt{b} (1+\cot [c+d x])^2 (b+a \tan [c+d x]) \right) \right) \right) + \\
 & \quad \left(1 / \left((a^2+b^2) (-1+\cot [c+d x])^2 (1+\cot [c+d x])^2 (b+a \tan [c+d x]) \right) \right) \\
 & \quad 4 a^4 b \cos [2(c+d x)] (a+b \cot [c+d x]) \csc [c+d x]^3 \\
 & \quad \left(\frac{4 (a^2-b^2) \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot [c+d x]}}{\sqrt{a}}\right]}{\sqrt{a} \sqrt{b}} + \sqrt{2} (2(a-b) \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\cot [c+d x]}\right] - \right. \\
 & \quad \left. 2(a-b) \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\cot [c+d x]}\right] + (a+b) \left(\log \left[1-\sqrt{2} \sqrt{\cot [c+d x]}\right] + \right. \right. \\
 & \quad \left. \left. \cot [c+d x] \right) - \log \left[1+\sqrt{2} \sqrt{\cot [c+d x]}\right] + \cot [c+d x] \right) \Bigg) \sec [c+d x] - \\
 & \quad \frac{1}{4 (a^2+b^2) (1+\cot [c+d x])^2 (b+a \tan [c+d x])} (4 a^5-4 a^3 b^2) (a+b \cot [c+d x]) \\
 & \quad \csc [c+d x]^2 \left(-8 \sqrt{a} \sqrt{b} \operatorname{ArcTan}\left[\frac{\sqrt{b} \sqrt{\cot [c+d x]}}{\sqrt{a}}\right] + \sqrt{2} \right. \\
 & \quad \left(-2(a+b) \operatorname{ArcTan}\left[1-\sqrt{2} \sqrt{\cot [c+d x]}\right] + 2(a+b) \operatorname{ArcTan}\left[1+\sqrt{2} \sqrt{\cot [c+d x]}\right] + \right. \\
 & \quad \left. (a-b) \left(\log \left[1-\sqrt{2} \sqrt{\cot [c+d x]}\right] + \cot [c+d x] \right) - \log \left[\right. \right. \\
 & \quad \left. \left. 1+\sqrt{2} \sqrt{\cot [c+d x]}\right] + \cot [c+d x] \right) \Bigg) \sec [c+d x]^2 \sin [2(c+d x)] \Bigg)
 \end{aligned}$$

Problem 88: Result unnecessarily involves imaginary or complex numbers.

$$\int (a + b \cot [c + d x])^n dx$$

Optimal (type 5, 167 leaves, 5 steps):

$$- \left(\left(b (a + b \cot [c + d x])^{1+n} \text{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{a + b \cot [c + d x]}{a - \sqrt{-b^2}} \right] \right) / \right. \\ \left. \left(2 \sqrt{-b^2} (a - \sqrt{-b^2}) d (1+n) \right) \right) + \\ \left(b (a + b \cot [c + d x])^{1+n} \text{Hypergeometric2F1} \left[1, 1+n, 2+n, \frac{a + b \cot [c + d x]}{a + \sqrt{-b^2}} \right] \right) / \\ \left(2 \sqrt{-b^2} (a + \sqrt{-b^2}) d (1+n) \right)$$

Result (type 5, 161 leaves):

$$\frac{1}{2 d n} i (a + b \cot [c + d x])^n \\ \left(\left(\frac{a + b \cot [c + d x]}{b (-i + \cot [c + d x])} \right)^{-n} \text{Hypergeometric2F1} \left[-n, -n, 1-n, -\frac{a + i b}{b (-i + \cot [c + d x])} \right] - \right. \\ \left. \left(\frac{a + b \cot [c + d x]}{b (i + \cot [c + d x])} \right)^{-n} \text{Hypergeometric2F1} \left[-n, -n, 1-n, \frac{-a + i b}{b (i + \cot [c + d x])} \right] \right)$$

Problem 89: Unable to integrate problem.

$$\int (a + b \cot [e + f x])^m (d \tan [e + f x])^n dx$$

Optimal (type 6, 193 leaves, 8 steps):

$$- \frac{1}{2 f (1-n)} \text{AppellF1} \left[1-n, -m, 1, 2-n, -\frac{b \cot [e + f x]}{a}, -i \cot [e + f x] \right] \\ \cot [e + f x] (a + b \cot [e + f x])^m \left(1 + \frac{b \cot [e + f x]}{a} \right)^{-m} (d \tan [e + f x])^n - \\ \frac{1}{2 f (1-n)} \text{AppellF1} \left[1-n, -m, 1, 2-n, -\frac{b \cot [e + f x]}{a}, i \cot [e + f x] \right] \\ \cot [e + f x] (a + b \cot [e + f x])^m \left(1 + \frac{b \cot [e + f x]}{a} \right)^{-m} (d \tan [e + f x])^n$$

Result (type 8, 25 leaves):

$$\int (a + b \cot [e + f x])^m (d \tan [e + f x])^n dx$$

Problem 91: Result more than twice size of optimal antiderivative.

$$\int \frac{1 - i \cot [c + d x]}{\sqrt{a + b \cot [c + d x]}} dx$$

Optimal (type 3, 45 leaves, 3 steps):

$$-\frac{2 i \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{\sqrt{a+i b} d}$$

Result (type 3, 128 leaves):

$$-\frac{1}{\sqrt{a+i b} d} i \operatorname{Log}\left[\frac{1}{\sqrt{a+i b}}\right. \\ \left.2\left(i b e^{2 i(c+d x)}+a\left(-1+e^{2 i(c+d x)}\right)+\sqrt{a+i b}\left(-1+e^{2 i(c+d x)}\right)\right) \sqrt{a+\frac{i b\left(1+e^{2 i(c+d x)}\right)}{-1+e^{2 i(c+d x)}}}\right]$$

Problem 93: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{A+B \cot [c+d x]}{(a+b \cot [c+d x])^2} dx$$

Optimal (type 3, 111 leaves, 3 steps):

$$\frac{\left(a^2 A-A b^2+2 a b B\right) x}{\left(a^2+b^2\right)^2}+\frac{A b-A B}{\left(a^2+b^2\right) d\left(a+b \cot [c+d x]\right)}- \\ \frac{\left(2 a A b-a^2 B+b^2 B\right) \operatorname{Log}[b \cos [c+d x]+a \sin [c+d x]]}{\left(a^2+b^2\right)^2 d}$$

Result (type 3, 352 leaves):

$$\frac{1}{2\left(a^2+b^2\right)^2 d\left(a+b \cot [c+d x]\right)} \\ \left(2 a^2 A b+2 A b^3-2 a^3 B-2 a b^2 B+2 a^3 A c-4 i a^2 A b c-2 a A b^2 c+2 i a^3 B c+\right. \\ \left.4 a^2 b B c-2 i a b^2 B c+2 a^3 A d x-4 i a^2 A b d x-2 a A b^2 d x+2 i a^3 B d x+4 a^2 b B d x-\right. \\ \left.2 i a b^2 B d x-2 i\left(-2 a A b+a^2 B-b^2 B\right) \operatorname{ArcTan}[\tan [c+d x]]\left(a+b \cot [c+d x]\right)-\right. \\ \left.2 a^2 A b \operatorname{Log}\left[\left(b \cos [c+d x]+a \sin [c+d x]\right)^2\right]+a^3 B \operatorname{Log}\left[\left(b \cos [c+d x]+a \sin [c+d x]\right)^2\right]-\right. \\ \left.a b^2 B \operatorname{Log}\left[\left(b \cos [c+d x]+a \sin [c+d x]\right)^2\right]+b \cot [c+d x]\right. \\ \left.\left(2(a-i b)^2(A+i B)(c+d x)+\left(-2 a A b+a^2 B-b^2 B\right) \operatorname{Log}\left[\left(b \cos [c+d x]+a \sin [c+d x]\right)^2\right]\right)\right)$$

Problem 94: Result unnecessarily involves complex numbers and more than twice size of optimal antiderivative.

$$\int \frac{A + B \cot [c + d x]}{(a + b \cot [c + d x])^3} dx$$

Optimal (type 3, 175 leaves, 4 steps):

$$\frac{(a^3 A - 3 a A b^2 + 3 a^2 b B - b^3 B) x}{(a^2 + b^2)^3} + \frac{A b - a B}{2 (a^2 + b^2) d (a + b \cot [c + d x])^2} + \frac{2 a A b - a^2 B + b^2 B}{(a^2 + b^2)^2 d (a + b \cot [c + d x])} - \frac{(3 a^2 A b - A b^3 - a^3 B + 3 a b^2 B) \operatorname{Log}[b \operatorname{Cos}[c + d x] + a \operatorname{Sin}[c + d x]]}{(a^2 + b^2)^3 d}$$

Result (type 3, 863 leaves):

$$\begin{aligned} & (b^2 (A b - a B) (A + B \cot [c + d x]) \operatorname{Csc}[c + d x]^2 (b \operatorname{Cos}[c + d x] + a \operatorname{Sin}[c + d x])) / \\ & \left(2 (-i a + b)^2 (i a + b)^2 d (a + b \cot [c + d x])^3 (B \operatorname{Cos}[c + d x] + A \operatorname{Sin}[c + d x]) \right) - \\ & \left((-a^3 A + 3 a A b^2 - 3 a^2 b B + b^3 B) (c + d x) (A + B \cot [c + d x]) \right. \\ & \quad \left. \operatorname{Csc}[c + d x]^2 (b \operatorname{Cos}[c + d x] + a \operatorname{Sin}[c + d x])^3 \right) / \\ & \left((-i a + b)^3 (i a + b)^3 d (a + b \cot [c + d x])^3 (B \operatorname{Cos}[c + d x] + A \operatorname{Sin}[c + d x]) \right) + \\ & \left((-3 i a^7 A b^3 + 3 a^6 A b^4 - 5 i a^5 A b^5 + 5 a^4 A b^6 - i a^3 A b^7 + a^2 A b^8 + i a A b^9 - A b^{10} + i a^8 b^2 B - \right. \\ & \quad \left. a^7 b^3 B - i a^6 b^4 B + a^5 b^5 B - 5 i a^4 b^6 B + 5 a^3 b^7 B - 3 i a^2 b^8 B + 3 a b^9 B) (c + d x) \right. \\ & \quad \left. (A + B \cot [c + d x]) \operatorname{Csc}[c + d x]^2 (b \operatorname{Cos}[c + d x] + a \operatorname{Sin}[c + d x])^3 \right) / \left((a - i b)^2 \right. \\ & \quad \left. (a + i b)^3 b^2 (-i a + b)^3 (i a + b)^3 d (a + b \cot [c + d x])^3 (B \operatorname{Cos}[c + d x] + A \operatorname{Sin}[c + d x]) \right) - \\ & \left(i (-3 a^2 A b + A b^3 + a^3 B - 3 a b^2 B) \operatorname{ArcTan}[\operatorname{Tan}[c + d x]] (A + B \cot [c + d x]) \right. \\ & \quad \left. \operatorname{Csc}[c + d x]^2 (b \operatorname{Cos}[c + d x] + a \operatorname{Sin}[c + d x])^3 \right) / \\ & \left((a^2 + b^2)^3 d (a + b \cot [c + d x])^3 (B \operatorname{Cos}[c + d x] + A \operatorname{Sin}[c + d x]) \right) + \\ & \left((-3 a^2 A b + A b^3 + a^3 B - 3 a b^2 B) (A + B \cot [c + d x]) \operatorname{Csc}[c + d x]^2 \right. \\ & \quad \left. \operatorname{Log}[(b \operatorname{Cos}[c + d x] + a \operatorname{Sin}[c + d x])^2] (b \operatorname{Cos}[c + d x] + a \operatorname{Sin}[c + d x])^3 \right) / \\ & \left(2 (a^2 + b^2)^3 d (a + b \cot [c + d x])^3 (B \operatorname{Cos}[c + d x] + A \operatorname{Sin}[c + d x]) \right) + \\ & \left((A + B \cot [c + d x]) \operatorname{Csc}[c + d x]^2 (b \operatorname{Cos}[c + d x] + a \operatorname{Sin}[c + d x])^2 \right. \\ & \quad \left. (3 a A b \operatorname{Sin}[c + d x] - 2 a^2 B \operatorname{Sin}[c + d x] + b^2 B \operatorname{Sin}[c + d x]) \right) / \\ & \left((-i a + b)^2 (i a + b)^2 d (a + b \cot [c + d x])^3 (B \operatorname{Cos}[c + d x] + A \operatorname{Sin}[c + d x]) \right) \end{aligned}$$

Problem 95: Result more than twice size of optimal antiderivative.

$$\int (a + b \cot [c + d x])^{5/2} (A + B \cot [c + d x]) dx$$

Optimal (type 3, 188 leaves, 10 steps):

$$\frac{(a - i b)^{5/2} (i A + B) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{d} - \frac{(a + i b)^{5/2} (i A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{d} - \frac{2 (2 a A b + a^2 B - b^2 B) \sqrt{a+b \cot [c+d x]}}{d} - \frac{2 (A b + a B) (a+b \cot [c+d x])^{3/2}}{3 d} - \frac{2 B (a+b \cot [c+d x])^{5/2}}{5 d}$$

Result (type 3, 505 leaves):

$$\left(i (a^3 A - 3 a A b^2 - 3 a^2 b B + b^3 B) \left(\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{\sqrt{a-i b}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{\sqrt{a+i b}} \right) \right. \\ \left. (a+b \cot [c+d x])^3 (A+B \cot [c+d x]) \operatorname{Sin}[c+d x]^4 \right) / \\ \left(d (b \operatorname{Cos}[c+d x] + a \operatorname{Sin}[c+d x])^3 (B \operatorname{Cos}[c+d x] + A \operatorname{Sin}[c+d x]) \right) + \\ \left(3 a^2 A b - A b^3 + a^3 B - 3 a b^2 B \right) \left(\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{\sqrt{a-i b}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{\sqrt{a+i b}} \right) \\ \left. (a+b \cot [c+d x])^3 (A+B \cot [c+d x]) \operatorname{Sin}[c+d x]^4 \right) / \\ \left(d (b \operatorname{Cos}[c+d x] + a \operatorname{Sin}[c+d x])^3 (B \operatorname{Cos}[c+d x] + A \operatorname{Sin}[c+d x]) \right) + \\ \left((a+b \cot [c+d x])^{5/2} (A+B \cot [c+d x]) \left(\frac{2}{15} (-35 a A b - 23 a^2 B + 18 b^2 B) - \right. \right. \\ \left. \left. \frac{2}{15} (5 A b^2 \operatorname{Cos}[c+d x] + 11 a b B \operatorname{Cos}[c+d x]) \operatorname{Csc}[c+d x] - \frac{2}{5} b^2 B \operatorname{Csc}[c+d x]^2 \right) \right. \\ \left. \operatorname{Sin}[c+d x]^3 \right) / \left(d (b \operatorname{Cos}[c+d x] + a \operatorname{Sin}[c+d x])^2 (B \operatorname{Cos}[c+d x] + A \operatorname{Sin}[c+d x]) \right)$$

Problem 96: Result more than twice size of optimal antiderivative.

$$\int (a+b \cot [c+d x])^{3/2} (A+B \cot [c+d x]) dx$$

Optimal (type 3, 150 leaves, 9 steps):

$$\frac{(a - i b)^{3/2} (i A + B) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{d} - \frac{(a + i b)^{3/2} (i A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{d} - \frac{2 (A b + a B) \sqrt{a+b \cot [c+d x]}}{d} - \frac{2 B (a+b \cot [c+d x])^{3/2}}{3 d}$$

Result (type 3, 441 leaves):

$$\left(i (a^2 A - A b^2 - 2 a b B) \left(\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{\sqrt{a-i b}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{\sqrt{a+i b}} \right) \right. \\ \left. (a+b \cot [c+d x])^2 (A+B \cot [c+d x]) \sin [c+d x]^3 \right) / \\ \left(d (b \cos [c+d x] + a \sin [c+d x])^2 (B \cos [c+d x] + A \sin [c+d x]) \right) + \\ \left(2 a A b + a^2 B - b^2 B \right) \left(\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{\sqrt{a-i b}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{\sqrt{a+i b}} \right) \\ \left. (a+b \cot [c+d x])^2 (A+B \cot [c+d x]) \sin [c+d x]^3 \right) / \\ \left(d (b \cos [c+d x] + a \sin [c+d x])^2 (B \cos [c+d x] + A \sin [c+d x]) \right) + \\ \left((a+b \cot [c+d x])^{3/2} (A+B \cot [c+d x]) \left(-\frac{2}{3} (3 A b + 4 a B) - \frac{2}{3} b B \cot [c+d x] \right) \sin [c+d x]^2 \right) / \\ \left(d (b \cos [c+d x] + a \sin [c+d x]) (B \cos [c+d x] + A \sin [c+d x]) \right)$$

Problem 98: Result more than twice size of optimal antiderivative.

$$\int (-a + b \cot [c+d x]) (a+b \cot [c+d x])^{5/2} dx$$

Optimal (type 3, 151 leaves, 10 steps):

$$-\frac{(i a - b) (a - i b)^{5/2} \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{d} + \frac{(a + i b)^{5/2} (i a + b) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{d} + \frac{2 b (a^2 + b^2) \sqrt{a+b \cot [c+d x]}}{d} - \frac{2 b (a+b \cot [c+d x])^{5/2}}{5 d}$$

Result (type 3, 479 leaves):

$$\begin{aligned}
 & \left((-a + b \cot [c + d x]) (a + b \cot [c + d x])^{5/2} \right. \\
 & \quad \left. \left(-\frac{4}{5} b (2 a^2 + 3 b^2) + \frac{4}{5} a b^2 \cot [c + d x] + \frac{2}{5} b^3 \operatorname{Csc} [c + d x]^2 \right) \sin [c + d x]^3 \right) / \\
 & \quad \left(d (-b \cos [c + d x] + a \sin [c + d x]) (b \cos [c + d x] + a \sin [c + d x])^2 \right) + \\
 & \quad \left((a^2 + b^2) (-a + b \cot [c + d x]) (a + b \cot [c + d x])^{5/2} \right. \\
 & \quad \left. \left(\left(i (a^2 - b^2) \left(\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}} \right]}{\sqrt{a-i b}} - \frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}} \right]}{\sqrt{a+i b}} \right) \sqrt{a+b \cot [c+d x]} \right) / \right. \right. \\
 & \quad \left. \left(\sqrt{\operatorname{Csc} [c+d x]} \sqrt{b \cos [c+d x] + a \sin [c+d x]} \right) + \right. \\
 & \quad \left. \left(2 a b \left(\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}} \right]}{\sqrt{a-i b}} + \frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}} \right]}{\sqrt{a+i b}} \right) \sqrt{a+b \cot [c+d x]} \right) / \right. \\
 & \quad \left. \left. \left(\sqrt{\operatorname{Csc} [c+d x]} \sqrt{b \cos [c+d x] + a \sin [c+d x]} \right) \right) \right) / \\
 & \quad \left. (d \operatorname{Csc} [c+d x]^{7/2} (-b \cos [c+d x] + a \sin [c+d x]) (b \cos [c+d x] + a \sin [c+d x])^{5/2}) \right)
 \end{aligned}$$

Problem 99: Result unnecessarily involves imaginary or complex numbers.

$$\int (-a + b \cot [c + d x]) (a + b \cot [c + d x])^{3/2} dx$$

Optimal (type 3, 408 leaves, 13 steps):

$$\frac{b (a^2 + b^2) \operatorname{ArcTanh} \left[\frac{\sqrt{a + \sqrt{a^2 + b^2}} - \sqrt{2} \sqrt{a + b \cot [c + d x]}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right]}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d} -$$

$$\frac{b (a^2 + b^2) \operatorname{ArcTanh} \left[\frac{\sqrt{a + \sqrt{a^2 + b^2}} + \sqrt{2} \sqrt{a + b \cot [c + d x]}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right]}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d} - \frac{2 b (a + b \cot [c + d x])^{3/2}}{3 d} +$$

$$\left(b (a^2 + b^2) \operatorname{Log} [a + \sqrt{a^2 + b^2} + b \cot [c + d x] - \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} \sqrt{a + b \cot [c + d x]}] \right) /$$

$$\left(2 \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} d \right) -$$

$$\left(b (a^2 + b^2) \operatorname{Log} [a + \sqrt{a^2 + b^2} + b \cot [c + d x] + \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} \sqrt{a + b \cot [c + d x]}] \right) /$$

$$\left(2 \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} d \right)$$

Result (type 3, 178 leaves):

$$\left((-a + b \cot [c + d x]) (a + b \cot [c + d x]) \left(3 i \sqrt{a - i b} (a^2 + b^2) \operatorname{ArcTanh} \left[\frac{\sqrt{a + b \cot [c + d x]}}{\sqrt{a - i b}} \right] - \right. \right.$$

$$\left. \left. 3 i \sqrt{a + i b} (a^2 + b^2) \operatorname{ArcTanh} \left[\frac{\sqrt{a + b \cot [c + d x]}}{\sqrt{a + i b}} \right] + 2 b (a + b \cot [c + d x])^{3/2} \right) \right.$$

$$\left. \operatorname{Sin}[c + d x]^2 \right) / (-3 b^2 d \operatorname{Cos}[c + d x]^2 + 3 a^2 d \operatorname{Sin}[c + d x]^2)$$

Problem 100: Result unnecessarily involves imaginary or complex numbers.

$$\int (-a + b \cot [c + d x]) \sqrt{a + b \cot [c + d x]} dx$$

Optimal (type 3, 422 leaves, 13 steps):

$$\frac{b \sqrt{a^2 + b^2} \operatorname{ArcTanh} \left[\frac{\sqrt{a + \sqrt{a^2 + b^2}} - \sqrt{2} \sqrt{a + b \cot [c + d x]}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right]}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d} - \frac{b \sqrt{a^2 + b^2} \operatorname{ArcTanh} \left[\frac{\sqrt{a + \sqrt{a^2 + b^2}} + \sqrt{2} \sqrt{a + b \cot [c + d x]}}{\sqrt{a - \sqrt{a^2 + b^2}}} \right]}{\sqrt{2} \sqrt{a - \sqrt{a^2 + b^2}} d} - \frac{2 b \sqrt{a + b \cot [c + d x]}}{d} - \left(b \sqrt{a^2 + b^2} \operatorname{Log} [a + \sqrt{a^2 + b^2} + b \cot [c + d x] - \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} \sqrt{a + b \cot [c + d x]}] \right) / \left(2 \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} d \right) + \left(b \sqrt{a^2 + b^2} \operatorname{Log} [a + \sqrt{a^2 + b^2} + b \cot [c + d x] + \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} \sqrt{a + b \cot [c + d x]}] \right) / \left(2 \sqrt{2} \sqrt{a + \sqrt{a^2 + b^2}} d \right)$$

Result (type 3, 158 leaves):

$$\left((-a + b \cot [c + d x]) \left(\frac{i (a^2 + b^2) \operatorname{ArcTanh} \left[\frac{\sqrt{a + b \cot [c + d x]}}{\sqrt{a - i b}} \right]}{\sqrt{a - i b}} - \frac{i (a^2 + b^2) \operatorname{ArcTanh} \left[\frac{\sqrt{a + b \cot [c + d x]}}{\sqrt{a + i b}} \right]}{\sqrt{a + i b}} + \frac{2 b \sqrt{a + b \cot [c + d x]}}{\sin [c + d x]} \right) \right) / (d (-b \cos [c + d x] + a \sin [c + d x]))$$

Problem 102: Result more than twice size of optimal antiderivative.

$$\int \frac{A + B \cot [c + d x]}{(a + b \cot [c + d x])^{3/2}} dx$$

Optimal (type 3, 138 leaves, 8 steps):

$$\frac{(i A + B) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{(a-i b)^{3/2} d} - \frac{(i A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{(a+i b)^{3/2} d} + \frac{2(A b - a B)}{(a^2 + b^2) d \sqrt{a+b \cot [c+d x]}}$$

Result (type 3, 476 leaves):

$$\begin{aligned} & (2(A + B \cot [c + d x]) \operatorname{Csc}[c + d x] \\ & \quad (b \cos [c + d x] + a \sin [c + d x]) (A b \sin [c + d x] - a B \sin [c + d x])) / \\ & \quad ((-i a + b) (i a + b) d (a + b \cot [c + d x])^{3/2} (B \cos [c + d x] + A \sin [c + d x])) + \\ & \quad \left((A + B \cot [c + d x]) \sqrt{\operatorname{Csc}[c + d x]} (b \cos [c + d x] + a \sin [c + d x])^{3/2} \right. \\ & \quad \left. \left(\left(i (a A + b B) \left(\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{\sqrt{a-i b}} - \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{\sqrt{a+i b}} \right) \sqrt{a+b \cot [c+d x]} \right) / \right. \right. \\ & \quad \left. \left(\sqrt{\operatorname{Csc}[c+d x]} \sqrt{b \cos [c+d x] + a \sin [c+d x]} \right) + \right. \\ & \quad \left. \left((-A b + a B) \left(\frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{\sqrt{a-i b}} + \frac{\operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{\sqrt{a+i b}} \right) \sqrt{a+b \cot [c+d x]} \right) / \right. \\ & \quad \left. \left. \left(\sqrt{\operatorname{Csc}[c+d x]} \sqrt{b \cos [c+d x] + a \sin [c+d x]} \right) \right) \right) / \\ & \quad \left. \left((a-i b) (a+i b) d (a+b \cot [c+d x])^{3/2} (B \cos [c+d x] + A \sin [c+d x]) \right) \right) \end{aligned}$$

Problem 103: Result more than twice size of optimal antiderivative.

$$\int \frac{A + B \cot [c + d x]}{(a + b \cot [c + d x])^{5/2}} dx$$

Optimal (type 3, 185 leaves, 9 steps):

$$\frac{(i A + B) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}}\right]}{(a-i b)^{5/2} d} - \frac{(i A - B) \operatorname{ArcTanh}\left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}}\right]}{(a+i b)^{5/2} d} + \frac{2(A b - a B)}{3(a^2 + b^2) d (a + b \cot [c + d x])^{3/2}} + \frac{2(2 a A b - a^2 B + b^2 B)}{(a^2 + b^2)^2 d \sqrt{a + b \cot [c + d x]}}$$

Result (type 3, 620 leaves):

$$\left((A + B \cot [c + d x]) \operatorname{Csc} [c + d x]^{3/2} (b \cos [c + d x] + a \sin [c + d x])^{5/2} \right. \\ \left. \left(\left(i (a^2 A - A b^2 + 2 a b B) \left(\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}} \right]}{\sqrt{a-i b}} - \frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}} \right]}{\sqrt{a+i b}} \right) \right. \right. \right. \\ \left. \left. \left. \sqrt{a+b \cot [c+d x]} \right) / \left(\sqrt{\operatorname{Csc} [c+d x]} \sqrt{b \cos [c+d x] + a \sin [c+d x]} \right) + \right. \right. \\ \left. \left(-2 a A b + a^2 B - b^2 B \right) \left(\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}} \right]}{\sqrt{a-i b}} + \frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}} \right]}{\sqrt{a+i b}} \right) \right. \\ \left. \left. \left. \sqrt{a+b \cot [c+d x]} \right) / \left(\sqrt{\operatorname{Csc} [c+d x]} \sqrt{b \cos [c+d x] + a \sin [c+d x]} \right) \right) \right) / \\ \left((a-i b)^2 (a+i b)^2 d (a+b \cot [c+d x])^{5/2} (B \cos [c+d x] + A \sin [c+d x]) \right) + \\ \left((A + B \cot [c+d x]) \operatorname{Csc} [c+d x]^2 (b \cos [c+d x] + a \sin [c+d x])^3 \right. \\ \left(-\frac{2 (A b - a B)}{3 (-i a + b)^2 (i a + b)^2} + \frac{2 b^2 (A b - a B)}{3 (-i a + b)^2 (i a + b)^2 (b \cos [c+d x] + a \sin [c+d x])^2} + \right. \\ \left. \left. \frac{2 (8 a A b \sin [c+d x] - 5 a^2 B \sin [c+d x] + 3 b^2 B \sin [c+d x])}{3 (-i a + b)^2 (i a + b)^2 (b \cos [c+d x] + a \sin [c+d x])} \right) \right) / \\ \left(d (a+b \cot [c+d x])^{5/2} (B \cos [c+d x] + A \sin [c+d x]) \right)$$

Problem 106: Result more than twice size of optimal antiderivative.

$$\int \frac{-a + b \cot [c + d x]}{(a + b \cot [c + d x])^{5/2}} dx$$

Optimal (type 3, 174 leaves, 9 steps):

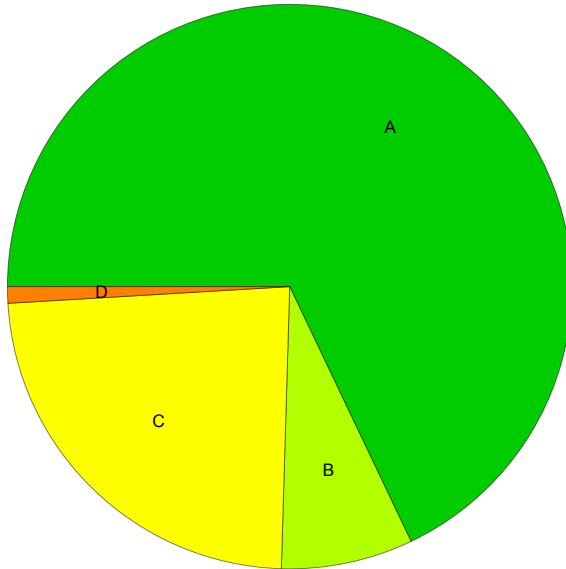
$$-\frac{(i a - b) \operatorname{ArcTanh} \left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a-i b}} \right]}{(a-i b)^{5/2} d} + \frac{(i a + b) \operatorname{ArcTanh} \left[\frac{\sqrt{a+b \cot [c+d x]}}{\sqrt{a+i b}} \right]}{(a+i b)^{5/2} d} - \\ \frac{4 a b}{3 (a^2 + b^2) d (a + b \cot [c + d x])^{3/2}} - \frac{2 b (3 a^2 - b^2)}{(a^2 + b^2)^2 d \sqrt{a + b \cot [c + d x]}}$$

Result (type 3, 587 leaves):

$$\left((-a + b \cot [c + d x]) \operatorname{Csc} [c + d x]^{3/2} (b \cos [c + d x] + a \sin [c + d x])^{5/2} \right. \\ \left. \left(\left(i (a^3 - 3 a b^2) \left(\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a + b \cot [c + d x]}}{\sqrt{a - i b}} \right]}{\sqrt{a - i b}} - \frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a + b \cot [c + d x]}}{\sqrt{a + i b}} \right]}{\sqrt{a + i b}} \right) \sqrt{a + b \cot [c + d x]} \right) / \right. \right. \\ \left. \left(\sqrt{\operatorname{Csc} [c + d x]} \sqrt{b \cos [c + d x] + a \sin [c + d x]} \right) + \right. \\ \left. \left((-3 a^2 b + b^3) \left(\frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a + b \cot [c + d x]}}{\sqrt{a - i b}} \right]}{\sqrt{a - i b}} + \frac{\operatorname{ArcTanh} \left[\frac{\sqrt{a + b \cot [c + d x]}}{\sqrt{a + i b}} \right]}{\sqrt{a + i b}} \right) \sqrt{a + b \cot [c + d x]} \right) / \right. \\ \left. \left. \left(\sqrt{\operatorname{Csc} [c + d x]} \sqrt{b \cos [c + d x] + a \sin [c + d x]} \right) \right) \right) / \\ \left((a - i b)^2 (a + i b)^2 d (a + b \cot [c + d x])^{5/2} (-b \cos [c + d x] + a \sin [c + d x]) \right) + \\ \left((-a + b \cot [c + d x]) \operatorname{Csc} [c + d x]^2 (b \cos [c + d x] + a \sin [c + d x])^3 \right. \\ \left. \left(-\frac{4 a b}{3 (-i a + b)^2 (i a + b)^2} + \frac{4 a b^3}{3 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])^2} - \right. \right. \\ \left. \left. \frac{2 (-13 a^2 b \sin [c + d x] + 3 b^3 \sin [c + d x])}{3 (-i a + b)^2 (i a + b)^2 (b \cos [c + d x] + a \sin [c + d x])} \right) \right) / \\ \left(d (a + b \cot [c + d x])^{5/2} (-b \cos [c + d x] + a \sin [c + d x]) \right)$$

Summary of Integration Test Results

106 integration problems



- A - 72 optimal antiderivatives
- B - 8 more than twice size of optimal antiderivatives
- C - 25 unnecessarily complex antiderivatives
- D - 1 unable to integrate problems
- E - 0 integration timeouts